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Rosario Fazio
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Disentangling unitary dynamics with classically simulable quantum circuits

[arXiv: 2410.09001]

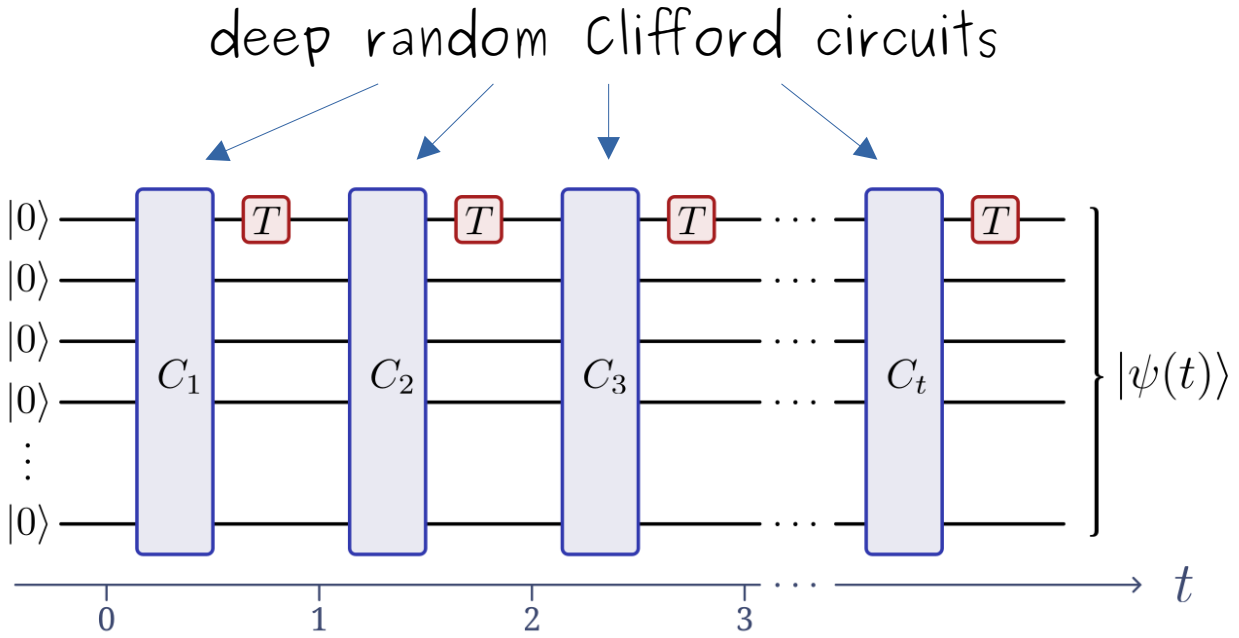
19th of November, 2024

Many-body quantum magic @ TII (Abu Dhabi)

Gerald E. Fux

The Abdus Salam International Centre for Theoretical Physics (ICTP)

Key result of [2410.09001]

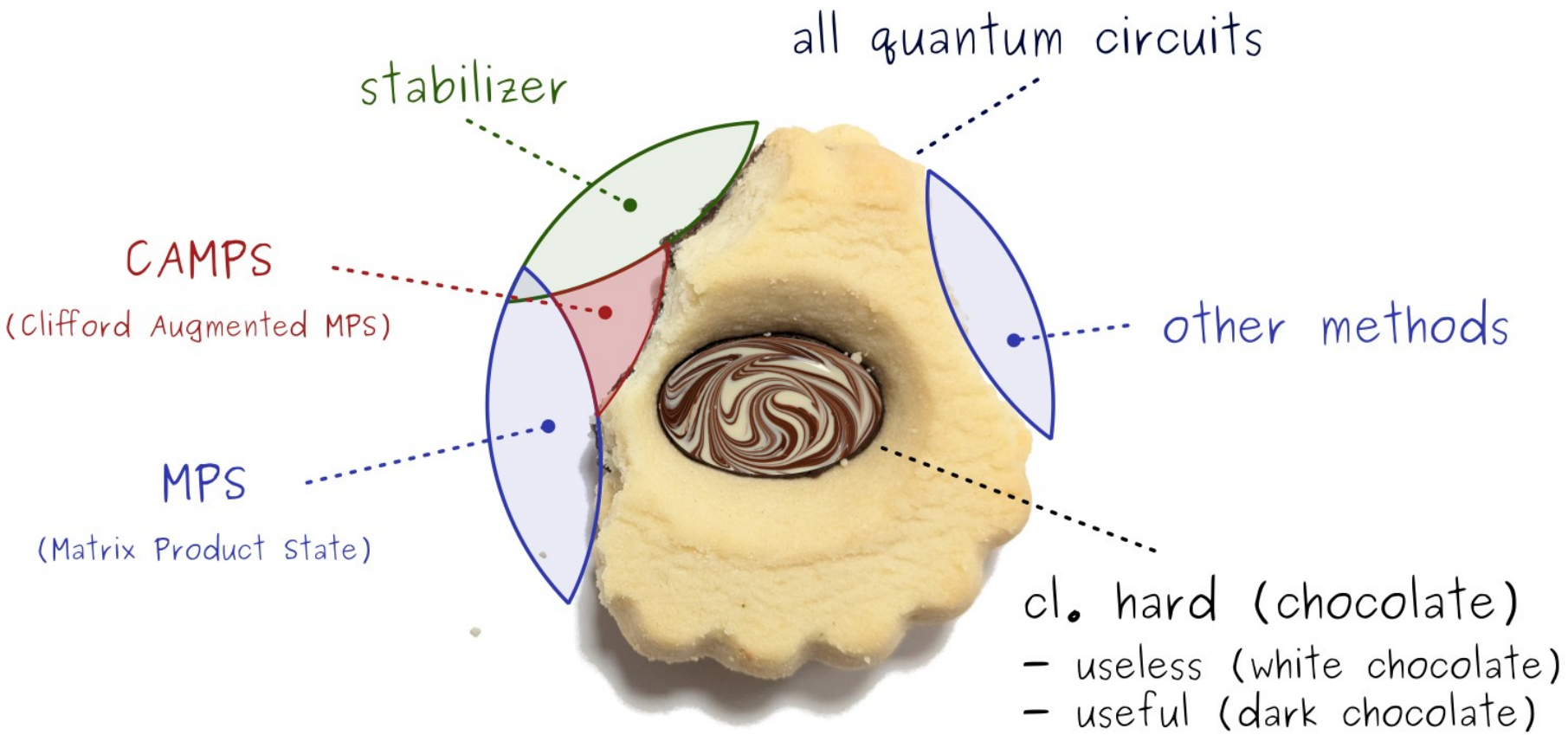


$$\langle\psi(t)|\, P\, |\psi(t)\rangle = \ldots \text{ can be classically simulated for } t \lesssim N$$

any Pauli string

number of Qbits

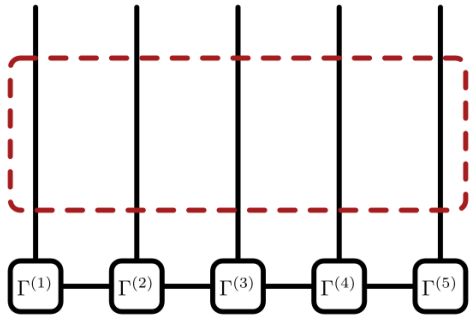
Quantum Advantage?



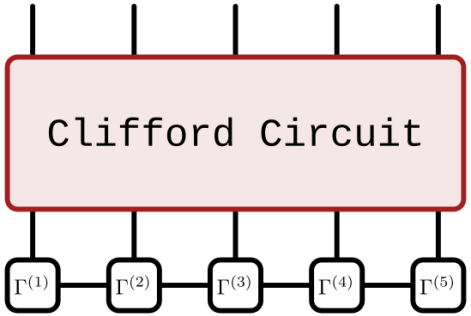
Outline

- **C**lifford **A**ugmented **M**atrix **P**roduct **S**tate
- Clifford + T-gate Circuit
  - ➔ Numerical results
  - ➔ Analytical considerations
- Future Directions

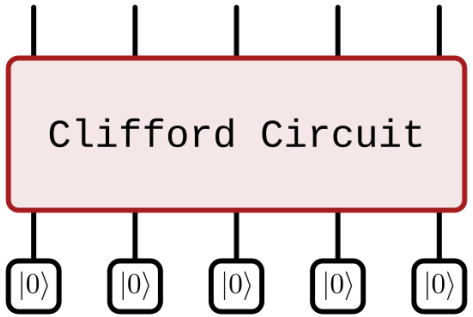
Clifford Augmented Matrix Product States



Matrix Product States



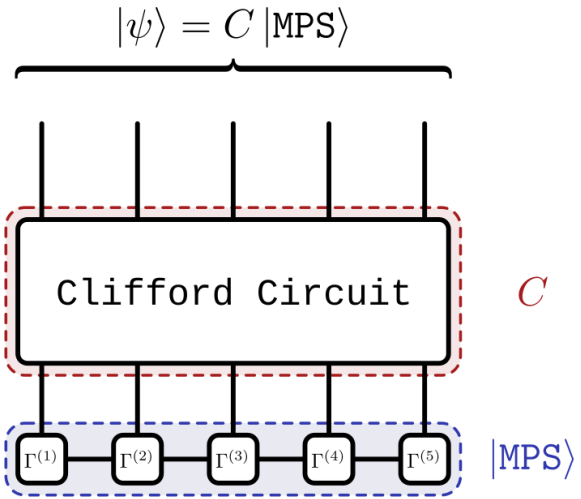
Clifford Augmented
Matrix Product States



Stabilizer States

[3] X Qian, J Huang, M Qin, *"Augmenting Density Matrix Renormalization Group with Clifford Circuits"*, PRL 133, 190402 (2024).

Clifford Augmented Matrix Product States



- Gate based evolution [1,2]
- Density matrix renormalization group (DMRG) [3]
- Time evolving variational principle (TDVP) [4,5]
- Gate based evolution [6, 7]

[1] S Masot-Llima, A Garcia-Saez, *"Stabilizer tensor networks: universal quantum simulator on a basis of ..."*, arXiv:2403.08724.
[2] AF Mello, A Santini, M Collura, *"Hybrid Stabilizer Matrix Product Operator"*, PRL 133, 150604 (2024)
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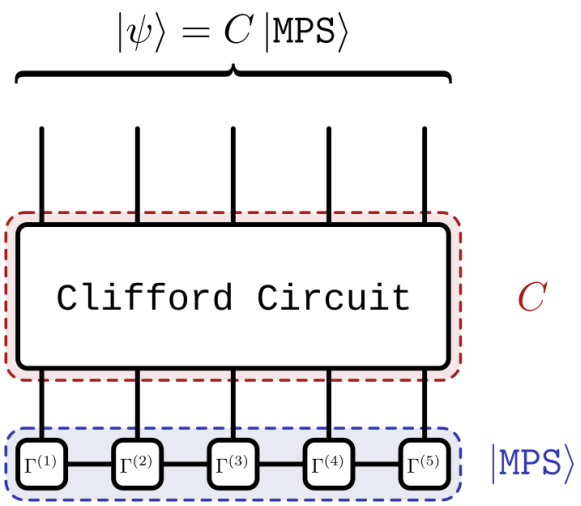
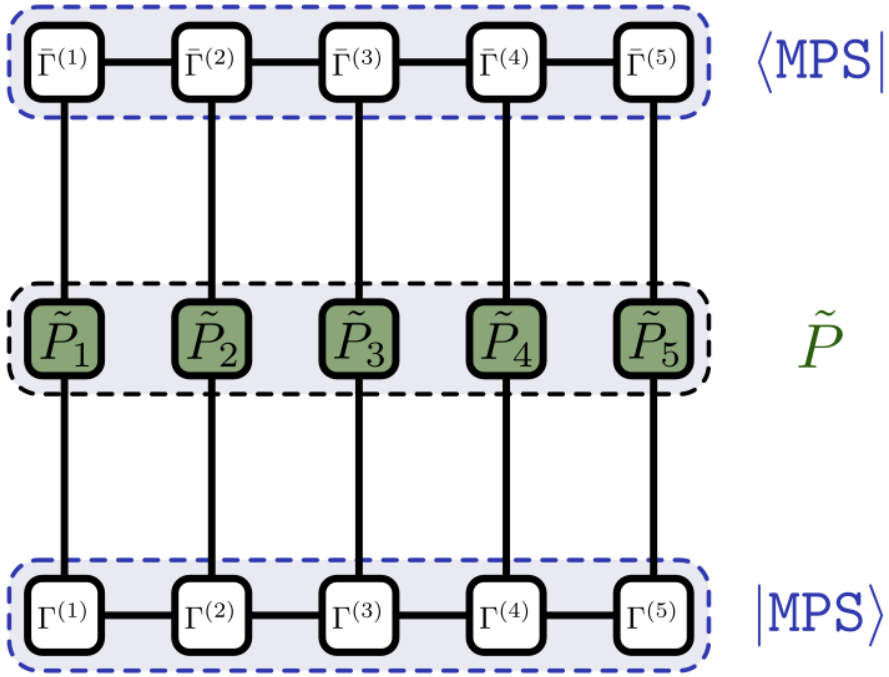


Pauli expectation

$$\langle \mathtt{MPS}|\, C^\dagger P C\, |\mathtt{MPS}\rangle = \langle \mathtt{MPS}|\, \tilde{P}\, |\mathtt{MPS}\rangle$$



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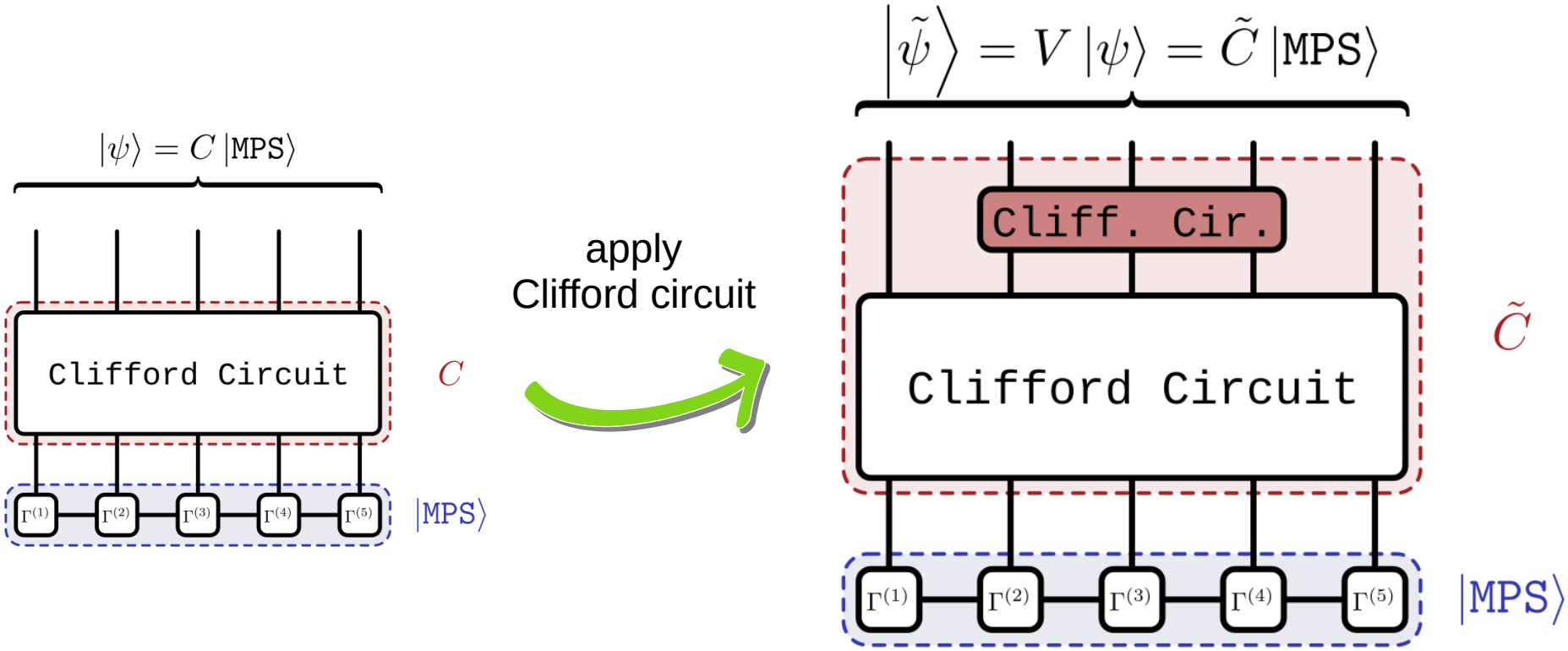

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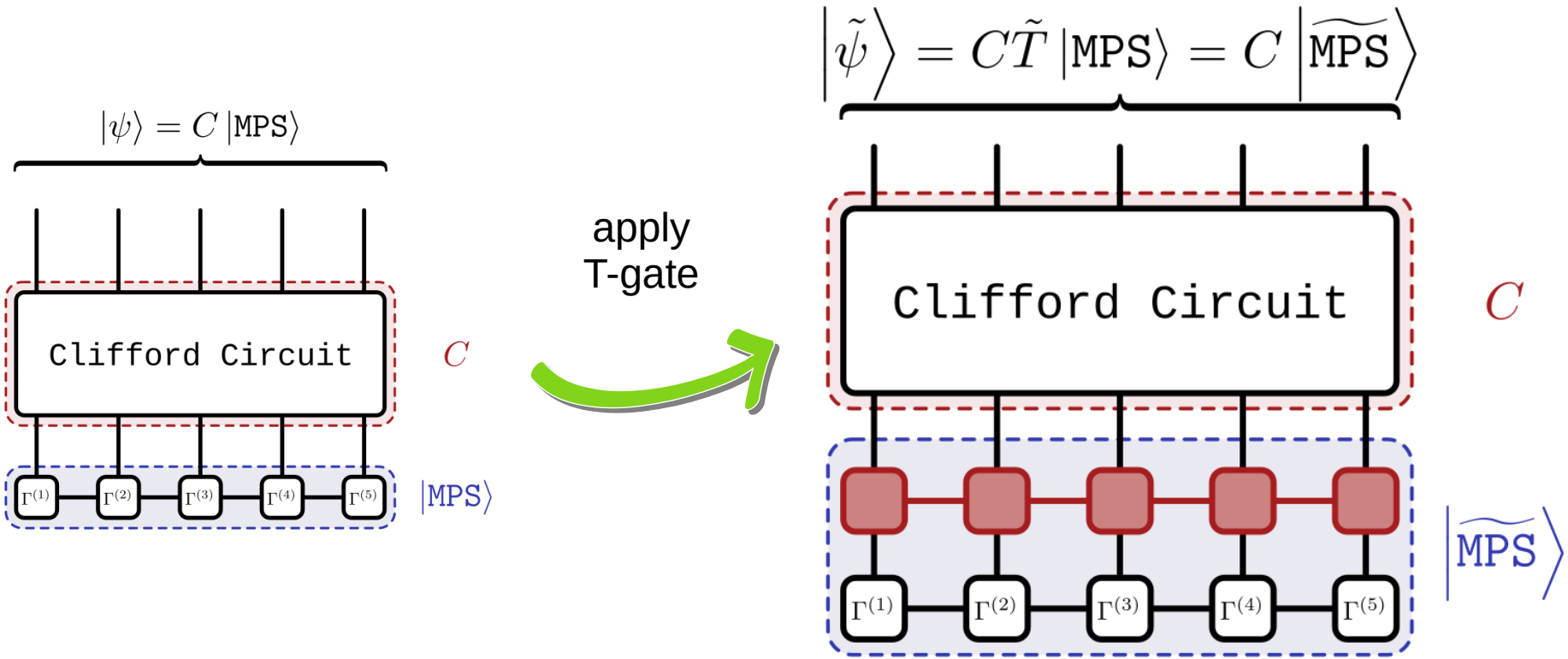

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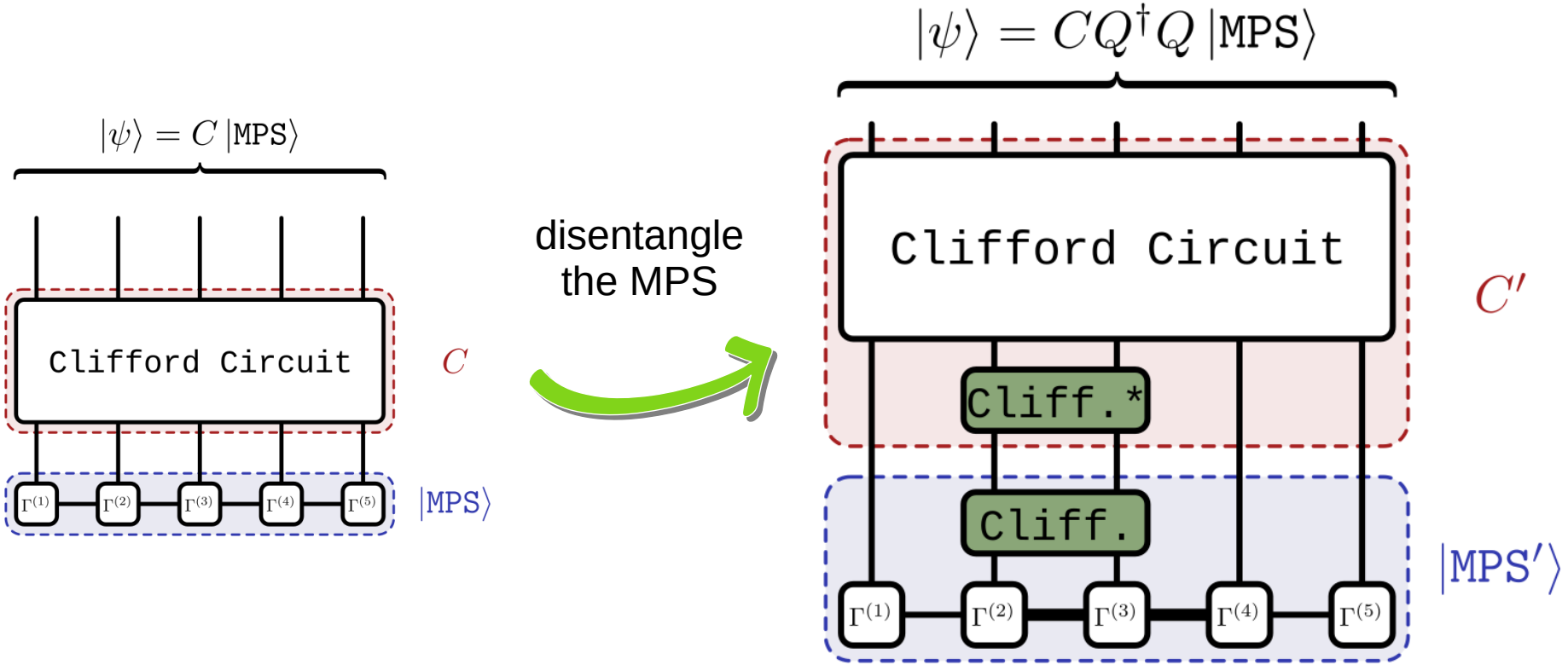
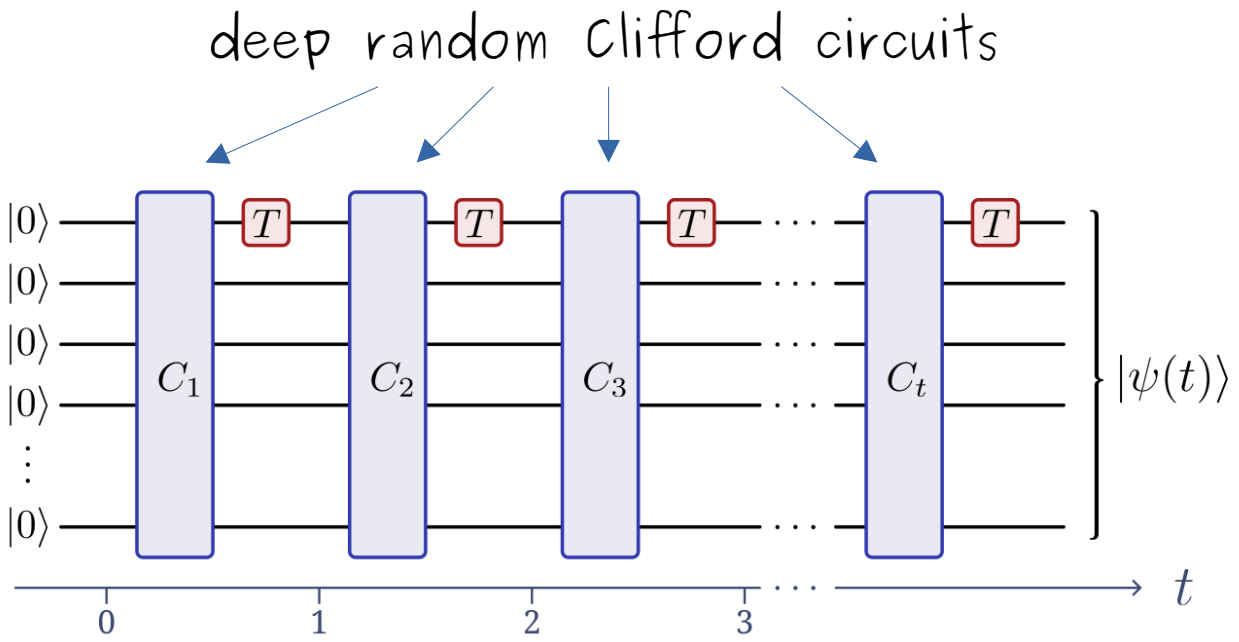


Clifford + T Circuit



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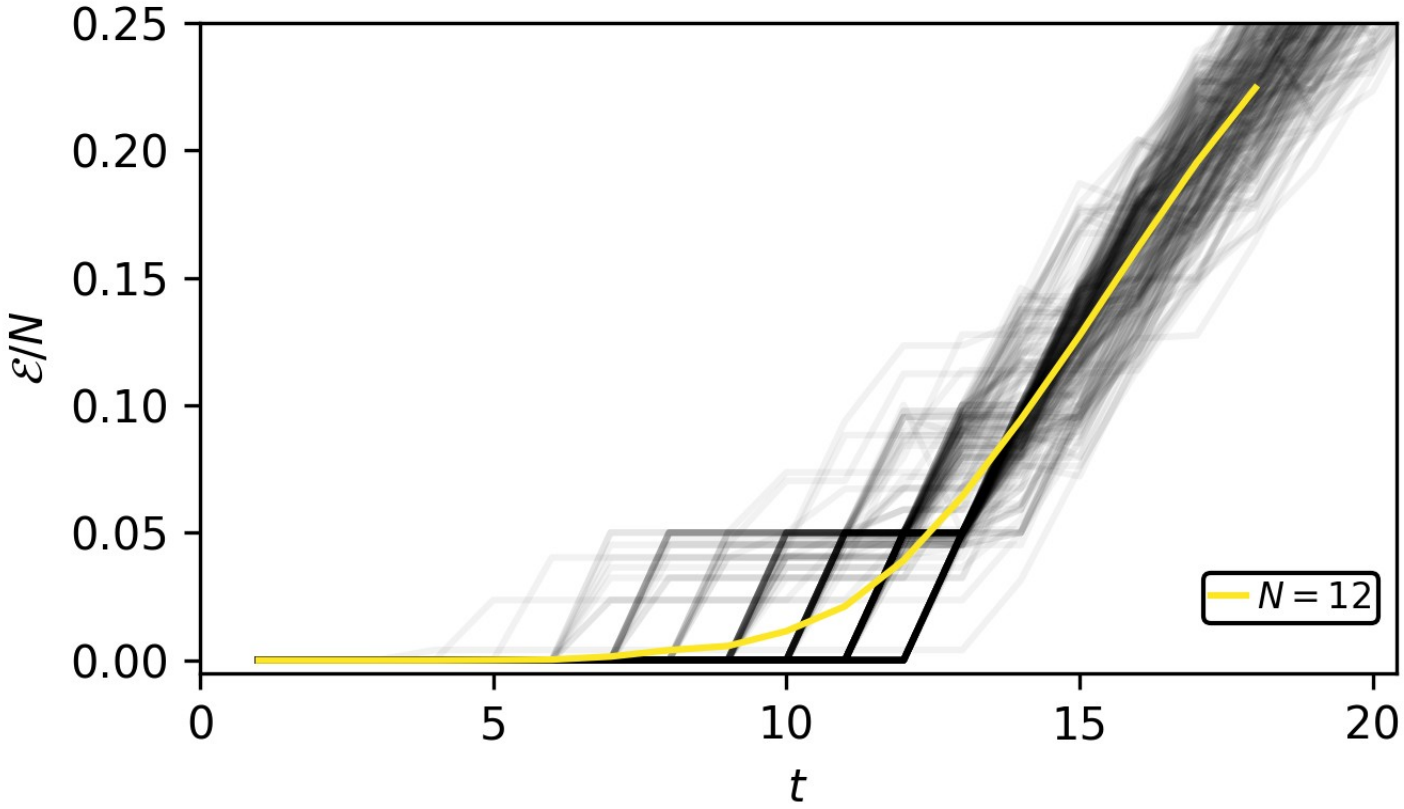

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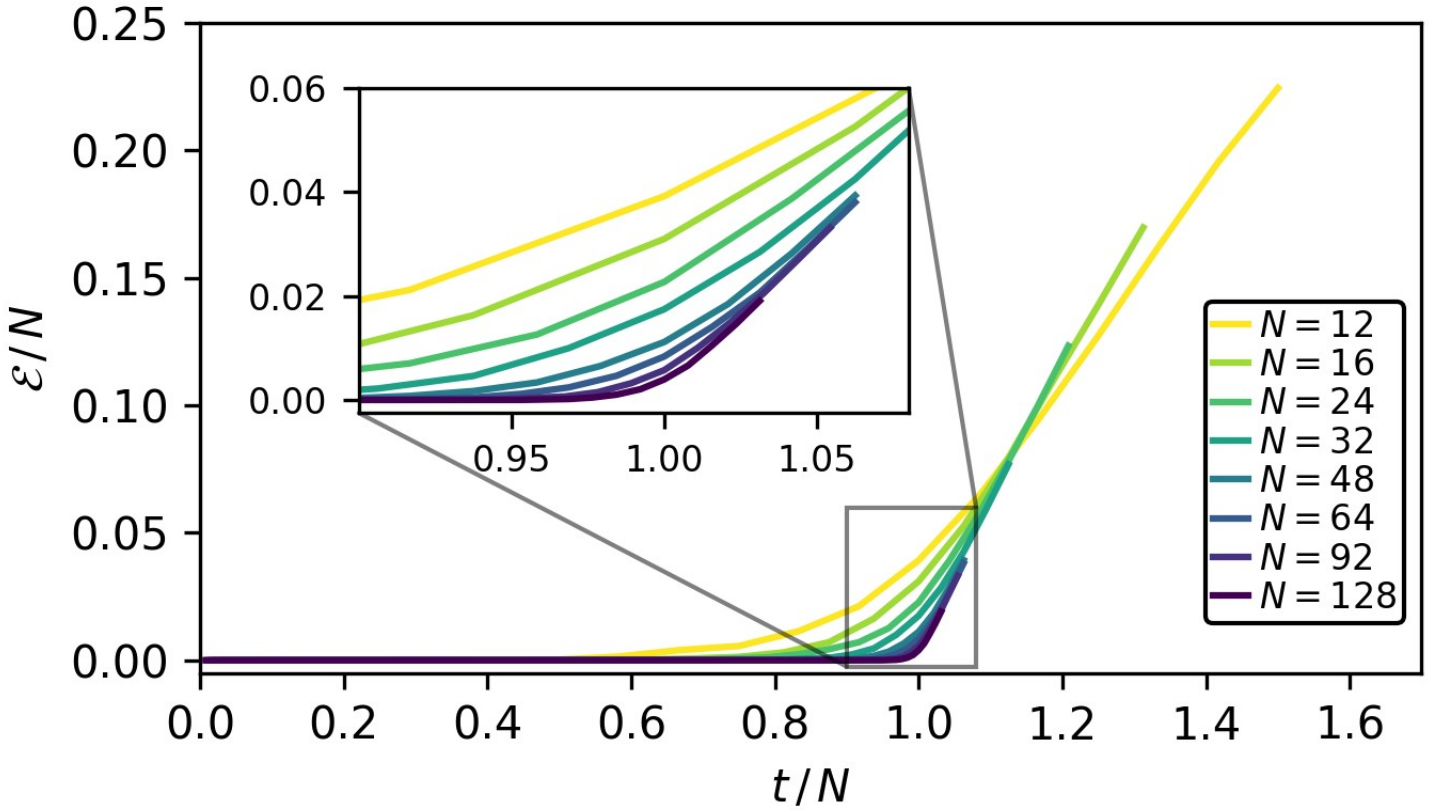

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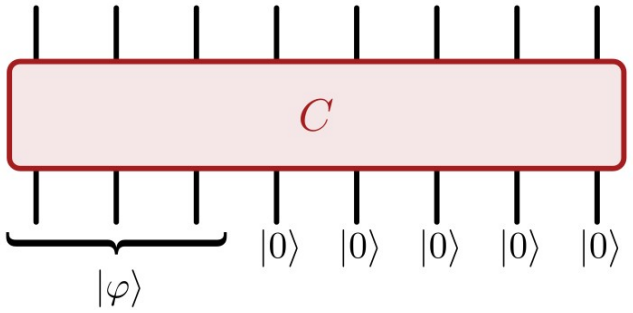

$$C\left(|\varphi\rangle\,|00000\rangle\right) \in \texttt{stab}\left[\langle \bar{Z}_4, \bar{Z}_5, \bar{Z}_6, \bar{Z}_7, \bar{Z}_8\rangle\right]$$

$$\bar{Z}_j := CZ_jC^\dagger\,, \quad \bar{X}_j := CX_jC^\dagger$$

$$|\varphi\rangle\,|00000\rangle \in \texttt{stab}\left[\langle Z_4, Z_5, Z_6, Z_7, Z_8\rangle\right]$$

$$\texttt{stab}[S] := \{|\psi\rangle \in \mathcal{H} |\, \forall P \in S : P\,|\psi\rangle = |\psi\rangle\}$$

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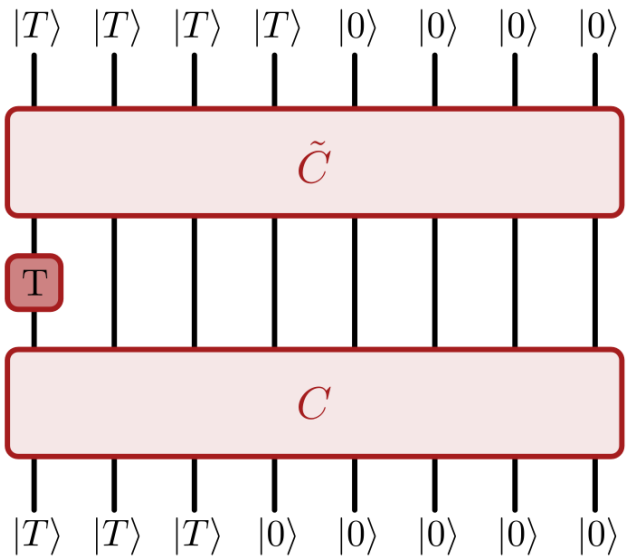
Clifford + T Circuit



$$|T\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$

if: T_1 does not act in the code space, i.e.

$$Z_1 \notin (\bar{\mathcal{P}}_3 \setminus \mathbb{1}) \otimes \langle \bar{Z}_4, \bar{Z}_5, \bar{Z}_6, \bar{Z}_7, \bar{Z}_8 \rangle$$

then:

$$\exists \tilde{C} \quad \text{such that } \tilde{C} T_1 C |\varphi\rangle |00000\rangle = |\varphi\rangle |T0000\rangle$$

Success for the (k+1)[th] T-gate:

$\mathcal{P}_3 \setminus \mathbb{1}$ $\qquad$ $\langle \bar{Z}_4, \bar{Z}_5, \bar{Z}_6, \bar{Z}_7, \bar{Z}_8 \rangle$

$$p_{k+1} = 1 - \frac{(4^k - 1)\, 2^{(N-k)}}{4^N - 1}$$

$\mathcal{P}_8 \setminus \mathbb{1}$

First fail after t* T-gates: $\langle t^* \rangle_{\text{analytic}} \approx N - 1.607$

$\langle t^* \rangle_{\text{numeric}} = N - (1.61 \pm 0.09)$

[7] GEF, B Béri, R Fazio, E Tirrito, *"Disentangling unitary dynamics with classically simulable quantum circuits"*, arXiv:2410.09001.

Future Direction: Quasi-Chotic to Chaotic

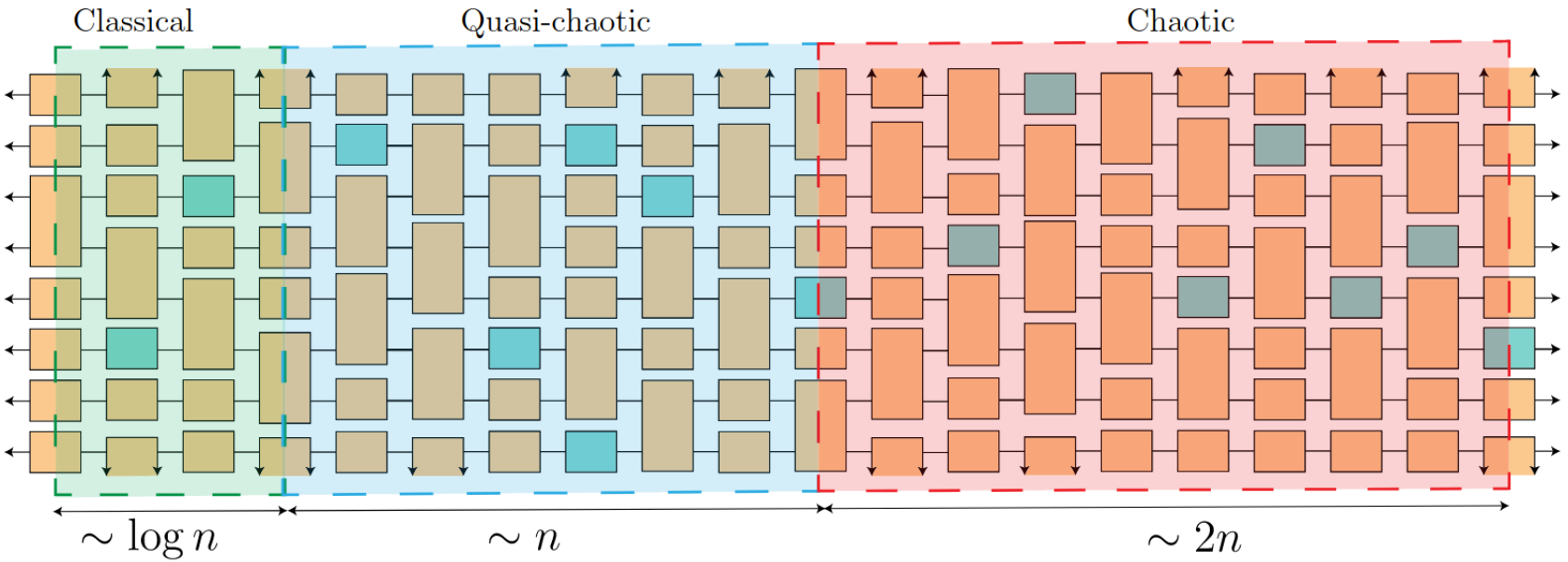
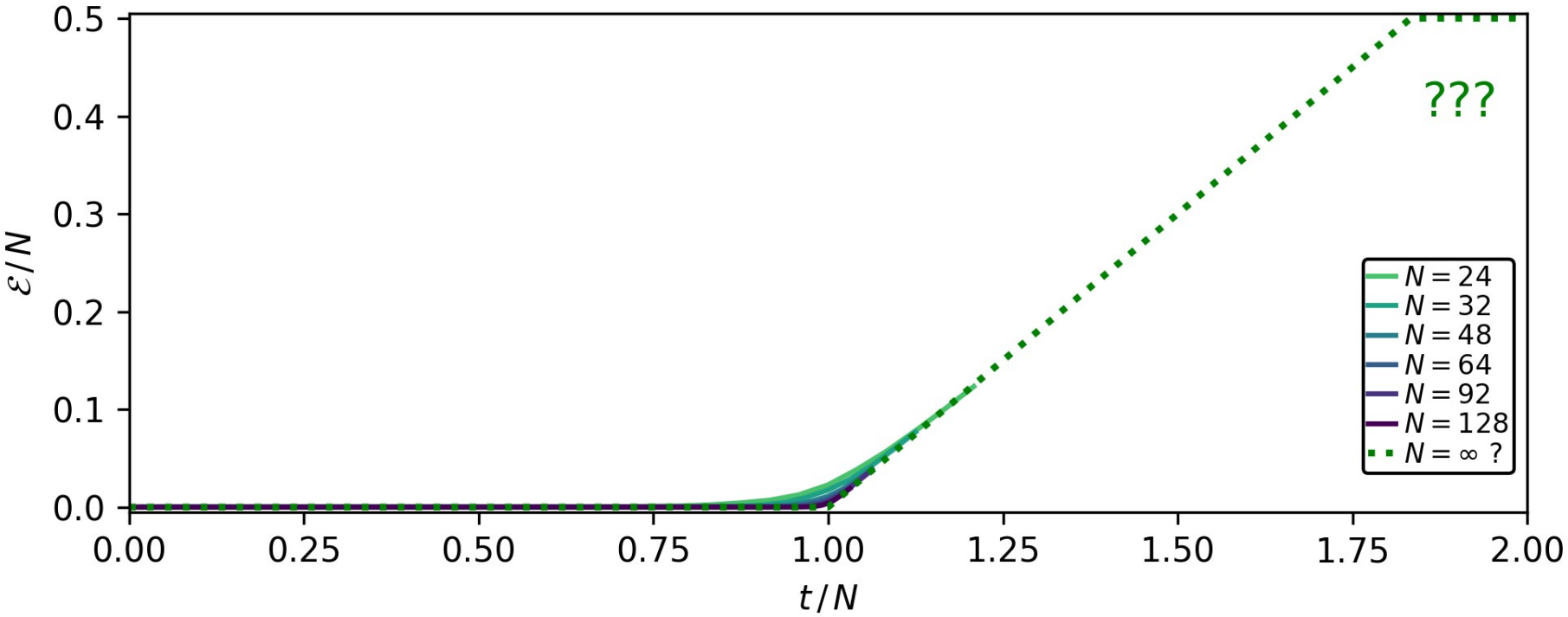


figure taken from [11]

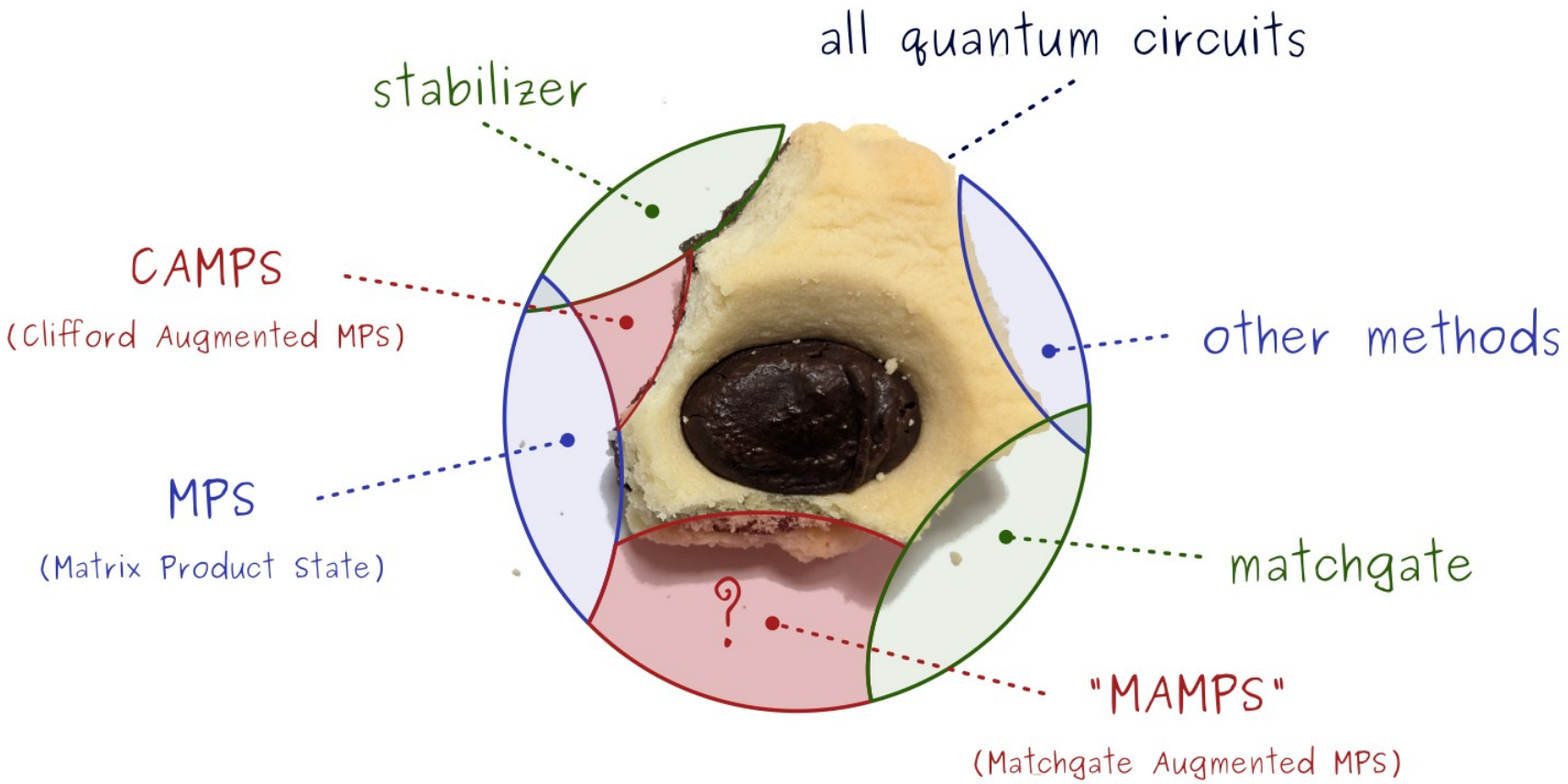
[8] C Chamon, A Hamma, and ER Mucciolo, *"Emergent irreversibility and entanglement spectrum statistics"*, PRL 112, 240501 (2014).
[9] D Shaffer, C Chamon, A Hamma, ER Mucciolo, *"Irreversibility and entanglement spectrum ..."*, J. Stat. Mech. P12007 (2014).
[10] S True, A Hamma, *"Transitions in Entanglement Complexity in Random Circuits"*, Quantum 6, 818 (2022).
[11] L Leone, SFE Oliviero, S Lloyd, A Hamma, *"Learning efficient decoders for quasichaotic quantum ..."*, PRA 109, 022429 (2024).

Future Direction: Quasi-Chaotic to Chaotic



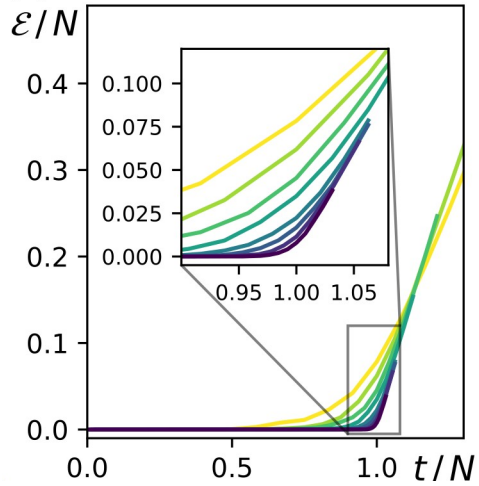
[7] GEF, B Béri, R Fazio, E Tirrito, *"Disentangling unitary dynamics with classically simulable quantum circuits"*, arXiv:2410.09001.

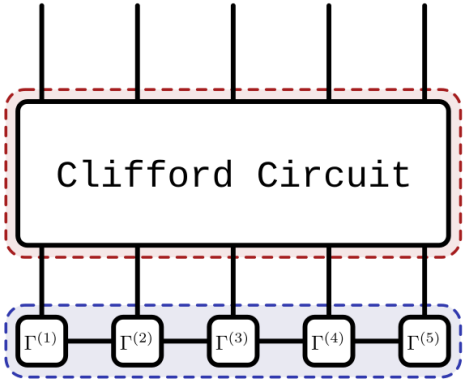
Future Direction: Matchgates

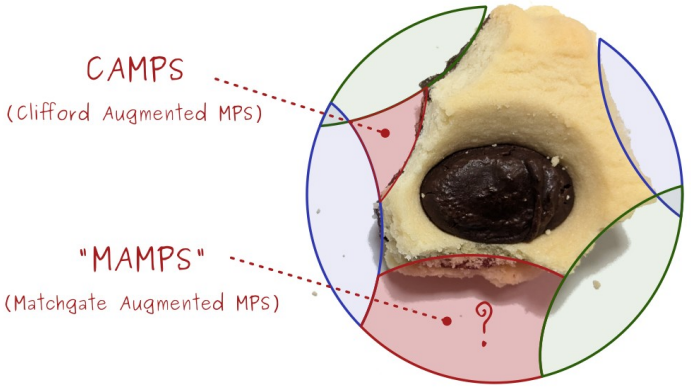


Summary

- **C**lifford **A**ugmented **MPS**
- Clifford + T-gate
- Hamiltonian Dynamics







Appendix

Bibliography

[0] L Leone, SF Oliviero, A Hamma, *"Stabilizer Rényi entropy"*, PRL 128(5), 050402 (2022).
[0] G Lami, M Collura, *"Nonstabilizerness via perfect pauli sampling of matrix product states"*, PRL 131, 180401 (2023).

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[9] D Shaffer, C Chamon, A Hamma, ER Mucciolo, *"Irreversibility and entanglement spectrum statistics in quantum circuits"*, J. Stat. Mech. P12007 (2014).
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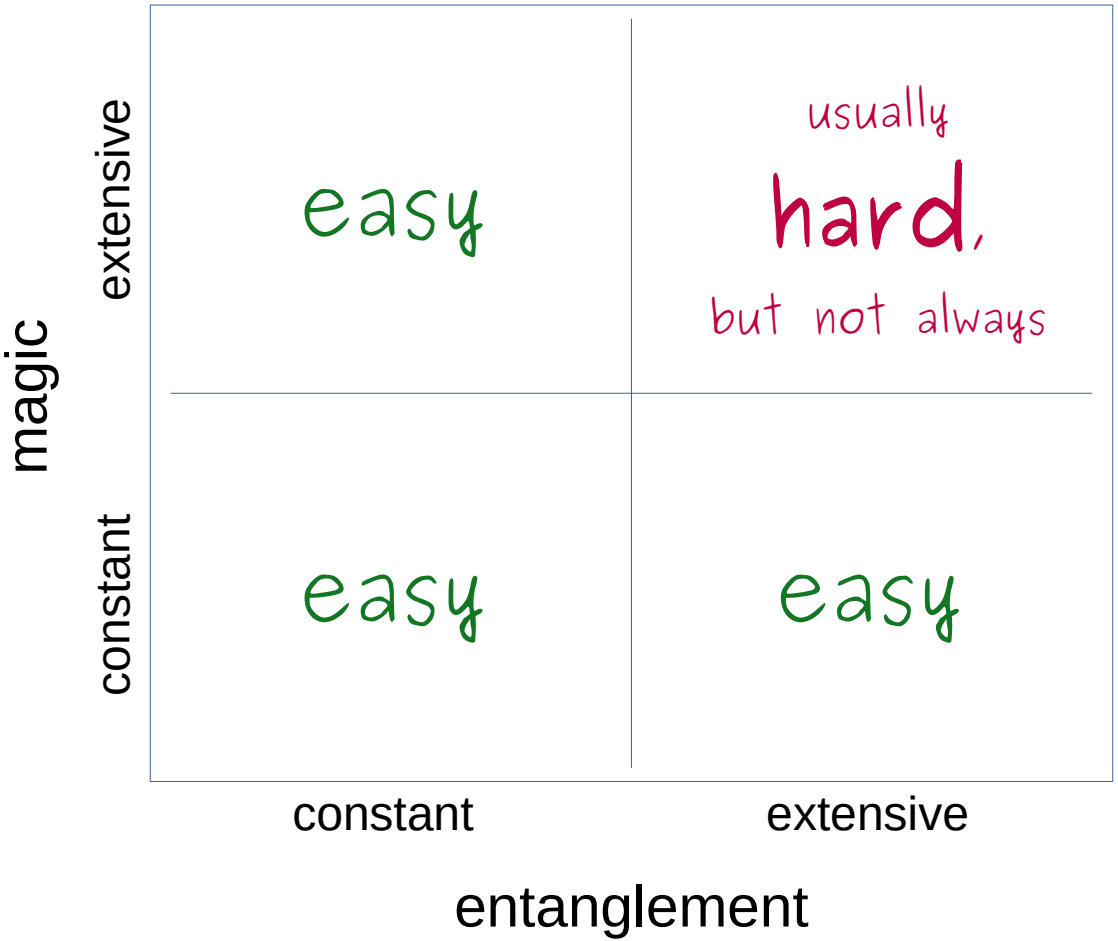
Quantum Advantage?



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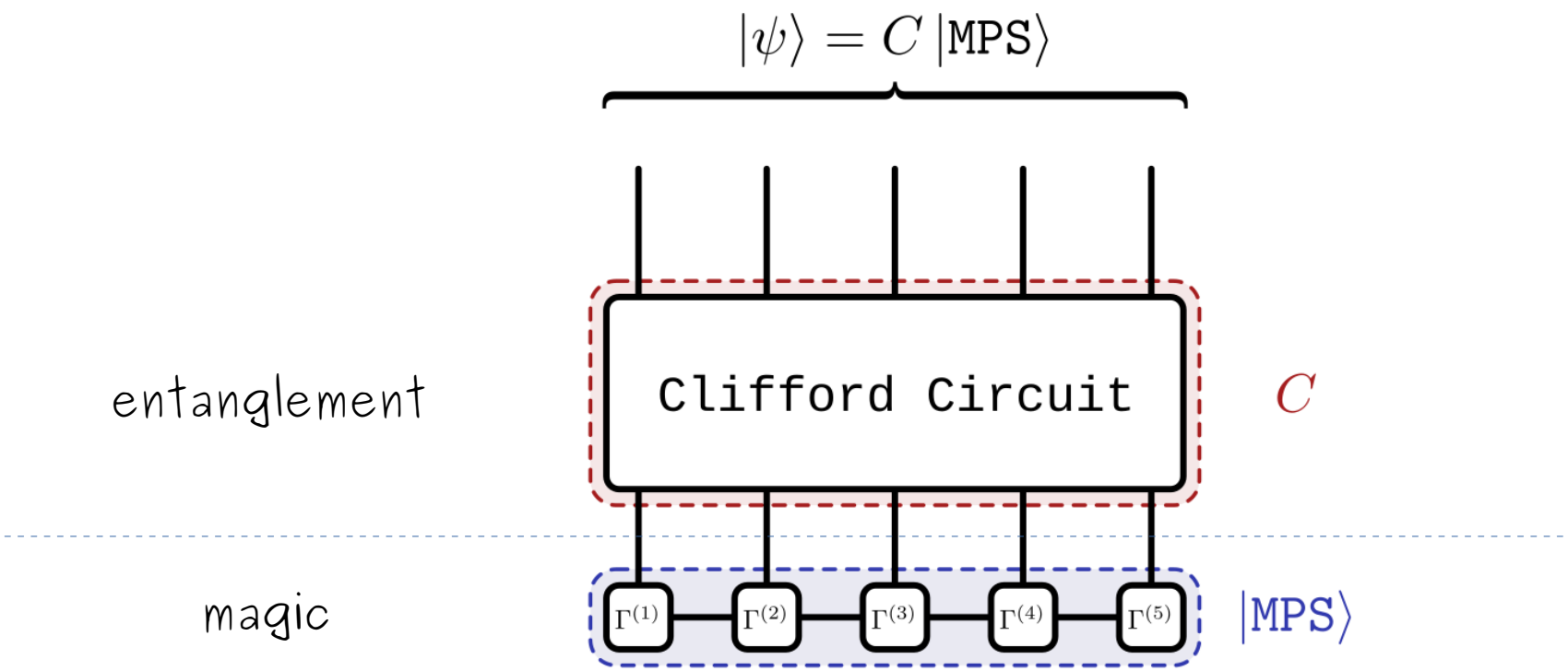
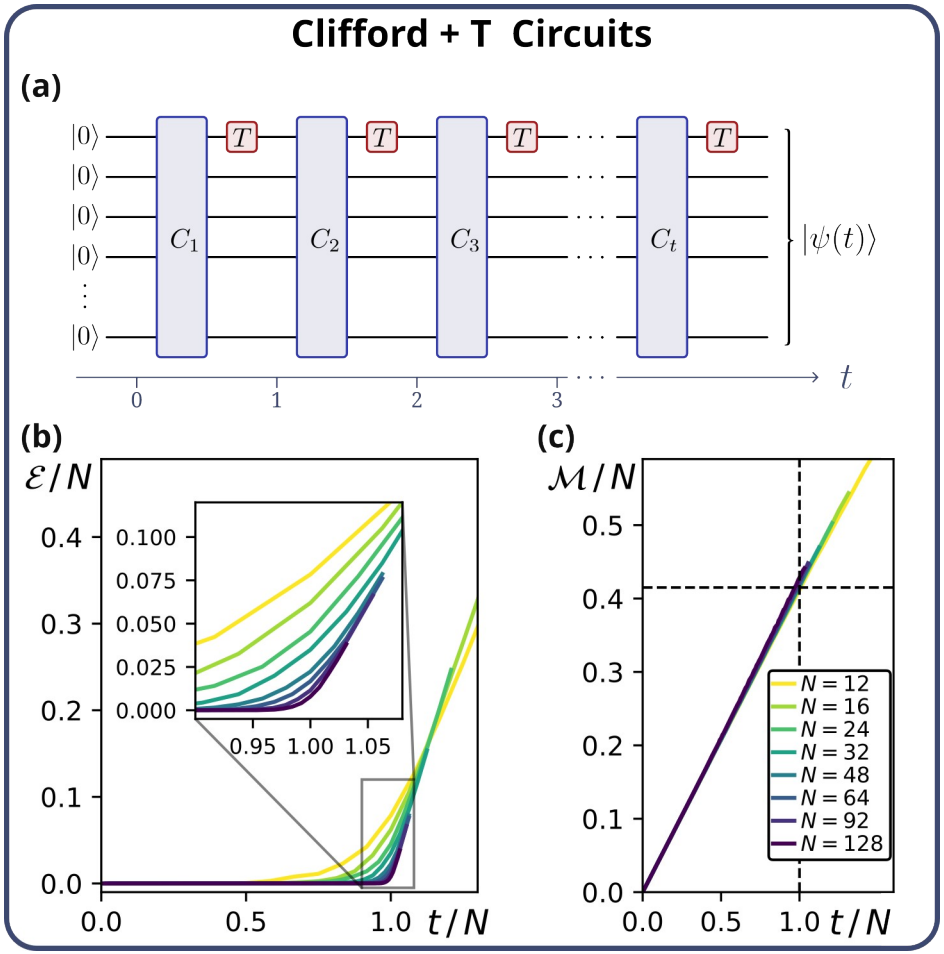
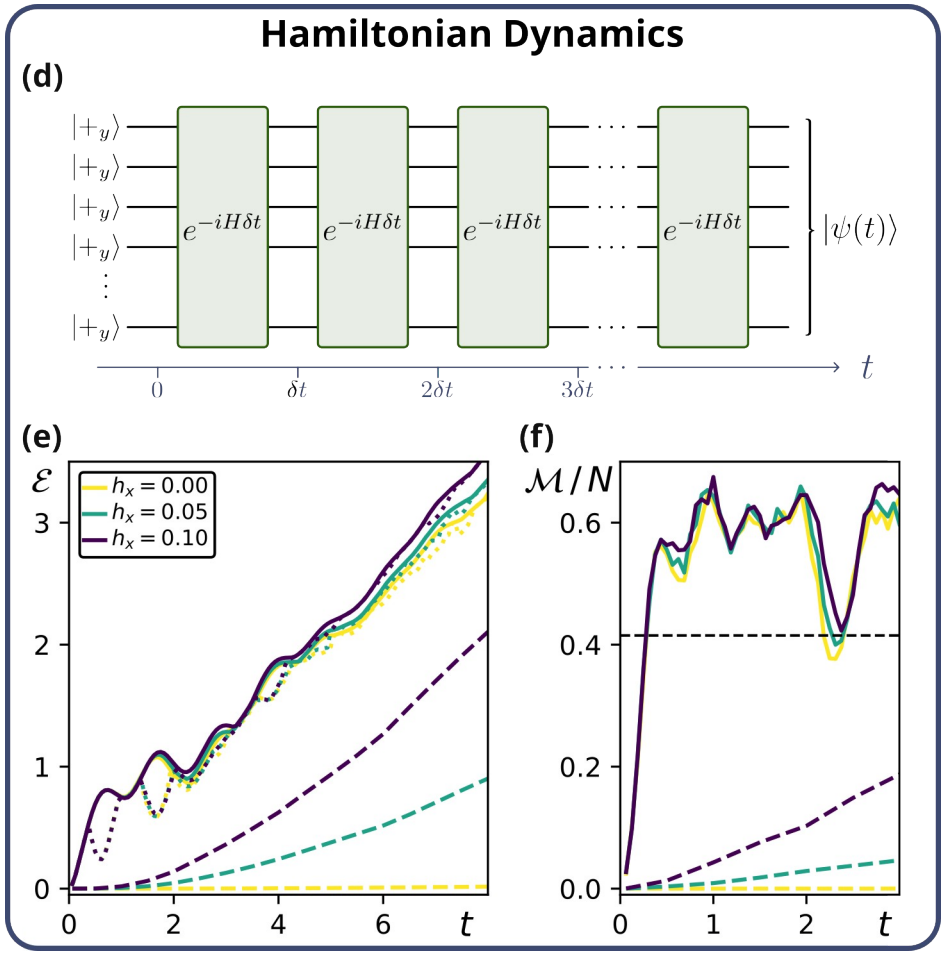


Hamiltonian Dynamics





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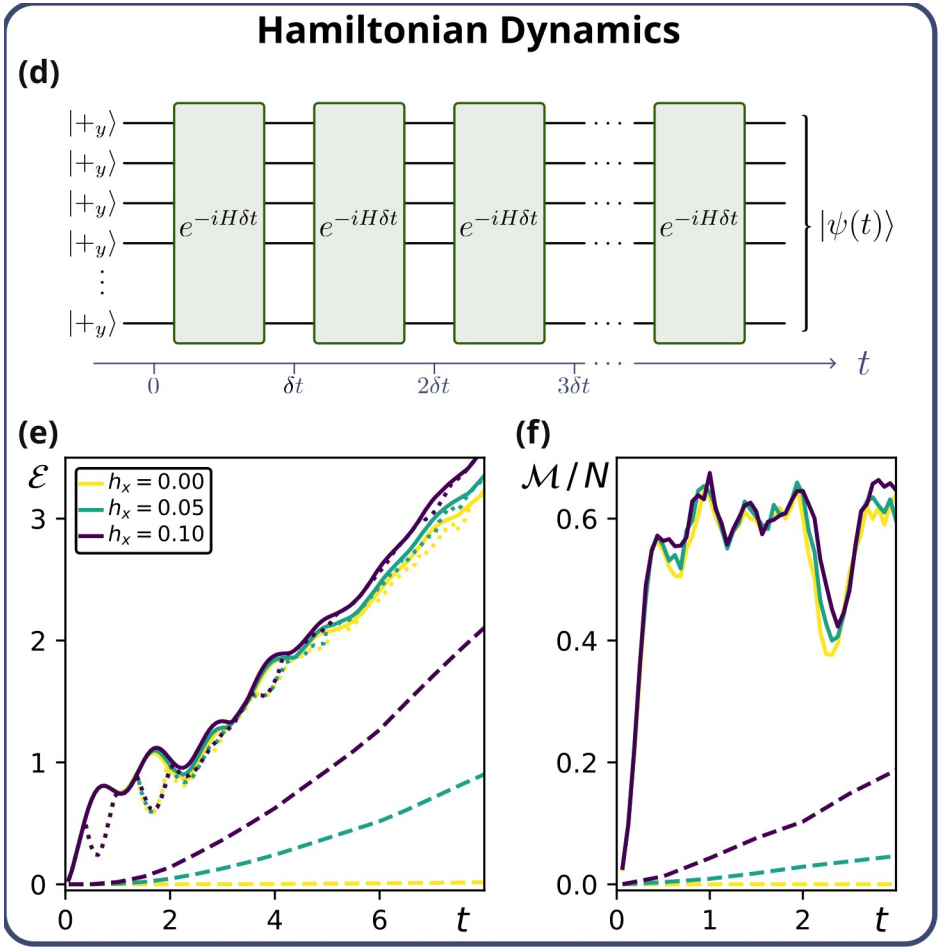
Hamiltonian Dynamics



$$H = \sum_{j=1}^{N} (h_x X_j + h_z Z_j) + \sum_{j=1}^{N-1} J X_j X_{j+1}$$

full state

disentangled with Clifford

disentangled with matchgate

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